தேசிய வெளிக்கள நிலையம் தொண்டைமானாறு

இரண்டாம் தவணைப் பரீட்சை - 2025

National Field Work Centre, Thondaimanaru.

2nd Term Examination - 2025

FWC

Gr : 12 (2026) இணைந்த கணிதம் புள்ளித்திட்டம்

1. $f(x) = 3x^2 - 6kx + 2k^2 + 2$

$= 3(x^2 - 2kx) + 2k^2 + 2$

$= 3(x-k)^2 + 2 - k^2$ (5)

$= 3(x-a)^2 + b$,

where $a = k$, $b = 2 - k^2$

$f(x) \geq 2 - k^2$ and '=' sign occurs when $x = k$.

$(f(x))_{min} = 2 - k^2$ (5)

$f(x) \geq k$ for all $x \in \mathbb{R}$

$\Leftrightarrow (f(x))_{min} = 2 - k^2 \geq k$ (5)

$\Leftrightarrow k^2 + k - 2 \leq 0$ (5)

$\Leftrightarrow (k+2)(k-1) \leq 0$

$\Leftrightarrow -2 \leq k \leq 1$ (5) $\boxed{25}$

2. $\frac{x^2+2}{x} < 3$

$\Leftrightarrow \frac{x^2+2}{x} - 3 < 0$

$\Leftrightarrow \frac{x^2-3x+2}{x} < 0$

$\Leftrightarrow \frac{(x-2)(x-1)}{x} < 0$ (5)

(5)	$x<0$	$0<x<1$	$1<x<2$	$x>2$
$x-2$	(−)	(−)	(−)	(+)
$x-1$	(−)	(−)	(+)	(+)
x	(−)	(+)	(+)	(+)
$\frac{(x-2)(x-1)}{x}$	(−)	(+)	(−)	(+)

$x < 0$ or $1 < x < 2$ (5)

$\frac{x^2+2}{x} > -3$

$\Leftrightarrow \frac{(-x)^2+2}{(-x)} < 3$ (5)

$\Leftrightarrow -x < 0$ or $1 < -x < 2$ (5) $\boxed{25}$

$\Leftrightarrow -2 < x < -1$ or $x > 0$

3. $\frac{x}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$ (5)

$x \equiv A(x-3) + B(x-2)$

Equating the coefficients of powers of x

x^1 : $1 = A + B$

x^0 : $0 = -3A - 2B$

$A = -2$, $B = 3$ (5)

$\frac{x}{(x-2)(x-3)} = \frac{-2}{x-2} + \frac{3}{x-3}$ (5)

$\frac{x^2-x+6}{(x-2)(x-3)} = \frac{x^2-5x+6}{(x-2)(x-3)} + \frac{4x}{(x-2)(x-3)}$ (5)

$= 1 - \frac{8}{x-2} + \frac{12}{x-3}$ (5) $\boxed{25}$

4. $\log_x \sqrt{3} - (\log_{\sqrt{3}} \sqrt{5}) \log_5 x = \frac{1}{2}$

$\log_x \sqrt{3} - \frac{1}{2}\log_{\sqrt{3}} 5 \cdot \frac{\log_{\sqrt{3}} x}{\log_{\sqrt{3}} 5} = \frac{1}{2}$ (5) (5)

$\frac{1}{\log_{\sqrt{3}} x} - \frac{1}{2}\log_{\sqrt{3}} x = \frac{1}{2}$

$\frac{1}{t} - \frac{1}{2}t = \frac{1}{2}$, where $t = \log_{\sqrt{3}} x$.

$t^2 + t - 2 = 0$ (5)

$(t+2)(t-1) = 0$

$t = -2$ or $t = 1$ (5)

$\log_{\sqrt3} x = -2$ or $\log_{\sqrt3} x = 1$

$x = \sqrt3^{\,-2}$ or $x = \sqrt3^{\,1}$

$x = \frac13$ or $x = \sqrt3$ (5) $\boxed{25}$

5. $\frac{1-\cos2x+\sin2x}{1+\cos2x+\sin2x}$

$= \frac{2\sin^2x + 2\sin x\cos x}{2\cos^2x + 2\sin x\cos x}$ (5) (5)

$= \frac{2\sin x(\sin x+\cos x)}{2\cos x(\cos x+\sin x)}$

$= \tan x.$ (5)

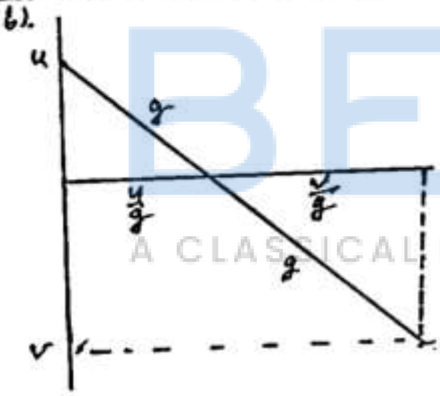
$x = 67\frac12^\circ$

$\tan 67\frac12^\circ = \frac{1-\cos135^\circ+\sin135^\circ}{1+\cos135^\circ+\sin135^\circ}$ (5)

$= \frac{1+\frac1{\sqrt2}+\frac1{\sqrt2}}{1-\frac1{\sqrt2}+\frac1{\sqrt2}}$

$= 1+\sqrt2.$ (5) $\boxed{25}$

6).



(5)

$\frac{u}{g}+\frac{v}{g} = 12$ (10)

$\frac12\left(\frac{v^2}{g}-\frac{u^2}{g}\right) = \frac{3u^2}{2g}$ (10)

$v^2-u^2 = 3u^2$

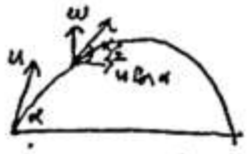
$v = 2u$

$\frac{u}{g}+\frac{2u}{g} = 12$

$u = 4g$

$\boxed{25}$

7.



(5)

$w = u\sin\alpha - gt$ (5)

$\tan\frac{\alpha}{2} = \frac{w}{u\cos\alpha}$ (5)

$= \frac{u\sin\alpha - gt}{u\cos\alpha}$

$gt = u\sin\alpha - u\cos\alpha\tan\frac{\alpha}{2}$

let $k = \tan\frac{\alpha}{2}$

$gt = u\frac{2k}{1+k^2} - u\frac{1-k^2}{1+k^2}\cdot k$ (5)

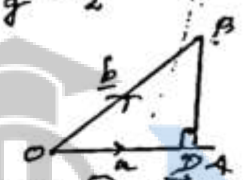
$= \frac{uk}{1+k^2}(2-1+k^2)$

$= uk$ (5)

$t = \frac{u}{g}\tan\frac{\alpha}{2}$

$\boxed{25}$

8.



let $\vec{OD} = \alpha\underline{a}$ (5), $\vec{BD} = \alpha\underline{a} - \underline{b}$ (5)

$\vec{OD}\cdot\vec{BD} = 0$

$\alpha\underline{a}\cdot(\alpha\underline{a} - \underline{b}) = 0$ (5)

$\alpha^2 a^2 - \alpha\underline{a}\cdot\underline{b} = 0$

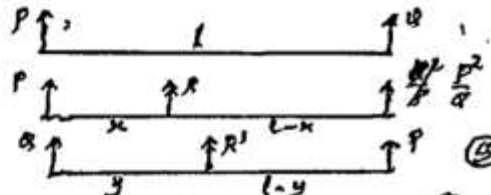
$\alpha(\alpha a^2 - \underline{a}\cdot\underline{b}) = 0$

$\alpha = \frac{\underline{a}\cdot\underline{b}}{a^2}$ (5)

$\vec{OD} = \left(\frac{\underline{a}\cdot\underline{b}}{a^2}\right)\underline{a}$ (5)

$\boxed{25}$

9).



(5)

$px = \frac{p^2}{q}(l-x)$ (5)

$qx = p(l-x) \Rightarrow (p+q)x = pl$ (5)

$qy = p(l-y) \Rightarrow (p+q)y = pl$ (5)

$\Rightarrow x = y$ (5)

$\boxed{25}$

11 (a)

$f(x) = ax^2 + 2bx + a = 0$

$ax^2 + 2bx + a \equiv \lambda(x-\alpha)(x-\beta)$ (5)

$ax^2 + 2bx + a \equiv \lambda(x^2 - (\alpha+\beta)x + \alpha\beta)$

$x^2: \; a = \lambda$

$x: \; 2b = -\lambda(\alpha+\beta)$ (10)

$x^0: \; a = \lambda\alpha\beta$

$\alpha+\beta = \frac{-2b}{a}$ (5)

$\alpha\beta = \frac{a}{a} = 1$ (5)

$\boxed{25}$

(i) $\Delta = 4b^2 - 4a(a)$ (5)

$= 4(b^2 - a^2)$

Since the roots are real and distinct

$\Delta > 0$ (5)

$\Leftrightarrow b^2 - a^2 > 0$

$\Leftrightarrow (a-b)(a+b) < 0$ (5)

$\Leftrightarrow -b < a < b$ (5) $[\because b > 0]$

$\boxed{20}$

Since α and β are the roots of $f(x)=0$

$a\alpha^2 + 2b\alpha + a = 0$ — (1) (5)

$a\beta^2 + 2b\beta + a = 0$ — (2)

(1) $\Rightarrow a(\alpha^2+1) = -2b\alpha$

$\Rightarrow \frac{\alpha^2+1}{\alpha} = -\frac{2b}{a}$ (5)

Similarly, $\frac{\beta^2+1}{\beta} = -\frac{2b}{a}$ (5)

$\therefore \frac{\alpha^2+1}{\alpha} = \frac{\beta^2+1}{\beta} = -\frac{2b}{a}$

$\boxed{15}$

$2\alpha - \beta + \frac{1}{\alpha}$

$= (\alpha - \beta) + \frac{\alpha^2+1}{\alpha}$ (5)

$= \alpha - \beta + (\alpha+\beta)$ (5) $\left[\because \frac{\alpha^2+1}{\alpha} = -\frac{2b}{a} = \alpha+\beta\right]$

$= 2\alpha$ (5)

Similarly, $2\beta - \alpha + \frac{1}{\beta} = 2\beta$ (5)

The equation whose roots are 2α and 2β

is $(x-2\alpha)(x-2\beta) = 0$ (5)

$x^2 - 2(\alpha+\beta)x + 4\alpha\beta = 0$ (5)

$x^2 - 2\left(\frac{-2b}{a}\right)x + 4(1) = 0$

The required equation is

$ax^2 + 4bx + 4a = 0$ (5)

$\boxed{35}$

(b) $g(x) = px^4 + 2x^3 + rx^2 + x - 2$

since $x-1$ is a factor of $g(x)$

$g(1) = 0$ (5)

$p + 2 + r + 1 - 2 = 0$ (5)

$p + 2 + r = 1$ — (1)

Since $g(x)$ is divided by $x+1$, the remainder is -4.

$g(-1) = -4$ (5)

$p - 2 + r - 1 - 2 = -4$ (5)

$p - 2 + r = -1$ — (2)

(1) $-$ (2) $\Rightarrow 2q = 2$ (5)

$q = 1$

$\boxed{25}$

Since $x+1 = (x-1)+2$ is a factor of $g(x-1)$

$g(-2) = 0$ (5)

$16p - 8q + 4r - 2 - 2 = 0$ (5)

$4p + r = 3$ — (3)

(1) $\Rightarrow p + r = 0$ — (4)

(3),(4) $\Rightarrow p = 1, r = -1$ (5)

$\boxed{15}$

$g(x) = x^4 + x^3 - x^2 + x - 2$

The remainder when $g(x)$ is divided by $x-2$ is $g(2)$

$g(2) = 16 + 8 - 4 + 2 - 2$ (5)

$= 20$

$g(x) = (x-1)(x^3 + 2x^2 + x + 2)$ (5)

$= (x-1)(x+2)(x^2+1)$ (5)

$\boxed{15}$

12. (a)

(i) $\lim_{x\to\infty} \frac{5-3x^2}{\sqrt{x^4+2}}$

$= \lim_{x\to\infty} \frac{\frac{5}{x^2}-3}{\sqrt{1+\frac{2}{x^4}}}$ (5)

$= \frac{0-3}{\sqrt{1+0}}$ (5)

$= -3.$ (5) $\boxed{15}$

(ii) $\lim_{x\to\sqrt{2}} \frac{x^4-4}{x^2+3\sqrt{2}x-8}$

$= \lim_{x\to\sqrt{2}} \frac{x^4-\sqrt{2}^4}{(x-\sqrt{2})(x+4\sqrt{2})}$ (5)

$= \lim_{x\to\sqrt{2}} \frac{x^4-\sqrt{2}^4}{x-\sqrt{2}} \lim_{x\to\sqrt{2}} \frac{1}{x+4\sqrt{2}}$ (5)

$= 4(\sqrt{2})^3 \times \frac{1}{5\sqrt{2}}$ (5) (5)

$= \frac{8}{5}.$ (5) $\boxed{25}$

(iii) $\lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{(4x^2-\pi^2)\cos x}$

$= \lim_{x\to\frac{\pi}{2}} \frac{1-\sin^2 x}{(4x^2-\pi^2)\cos x(1+\sin x)}$ (5)

$= \lim_{x\to\frac{\pi}{2}} \frac{\cos x}{4(x-\frac{\pi}{2})(x+\frac{\pi}{2})(1+\sin x)}$ (5)

$= \lim_{(\frac{\pi}{2}-x)\to 0} \frac{\sin(\frac{\pi}{2}-x)}{\frac{\pi}{2}-x} \cdot \lim_{x\to\frac{\pi}{2}} \frac{1}{-4(x+\frac{\pi}{2})(1+\sin x)}$ (5)

$= 1 \times \frac{1}{-4(\pi)(2)}$ (5) (5)

$= -\frac{1}{8\pi}.$ (5) $\boxed{30}$

(b) (i) Let $y = e^{3x}(\ln 2x)^2$

$\frac{dy}{dx} = e^{3x}\, 2(\ln 2x)\frac{1}{2x}(2) + (\ln 2x)^2 e^{3x}\cdot 3$ (10) (10)

$= e^{3x}\ln 2x\left(\frac{2}{x}+3\ln 2x\right)$

$= \frac{1}{x}e^{3x}\ln 2x\,(2+3x\ln 2x).$ $\boxed{20}$

(ii) Let $y = (\cos^2 2x)\, e^{\sin^{-1}4x}$

$\frac{dy}{dx} = \cos^2 2x\; e^{\sin^{-1}4x}\frac{1}{\sqrt{1-16x^2}}(4) + e^{\sin^{-1}4x}\, 2\cos 2x \sin 2x\,(2)$ (10) (10)

$\boxed{20}$

(c) $x = 2\sec^3\theta$

$\frac{dx}{d\theta} = 2(3)\sec^2\theta\sec\theta\tan\theta$ (10)

$= 6\sec^3\theta\tan\theta$

$y = 2\tan^3\theta$

$\frac{dy}{d\theta} = 2(3)\tan^2\theta\sec^2\theta$ (10)

$= 6\tan^2\theta\sec^2\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta}\times\frac{d\theta}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{6\tan^2\theta\sec^2\theta}{6\sec^3\theta\tan\theta}$ (5)

$\frac{dy}{dx} = \frac{\tan\theta}{\sec\theta} = \sin\theta$ (5)

$\frac{d^2y}{dx^2} = \cos\theta\frac{d\theta}{dx} = \cos\theta\,\frac{1}{6\sec^3\theta\tan\theta}$ (5)

$\left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{4}} = \frac{1}{\sqrt{2}}\,\frac{1}{6\sqrt{2}^3(1)} = \frac{1}{24}.$ (5) $\boxed{40}$

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13. (a)

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ (5)

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ (5) $\boxed{10}$

$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$ (5)

$= \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$ (5)

$= \frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}}$ (5)

$= \frac{\tan A + \tan B}{1-\tan A\tan B}$ (5) $\boxed{20}$

$\tan 2A = \tan(A+A)$

$= \frac{\tan A + \tan A}{1-\tan A\tan A}$ (5)

$= \frac{2\tan A}{1-\tan^2 A}$ (5) $\boxed{10}$

$$\tan 30^\circ = \frac{2\tan 15^\circ}{1-\tan^2 15^\circ} \quad (5)$$

$$\frac{1}{\sqrt{3}} = \frac{2\tan 15^\circ}{1-\tan^2 15^\circ}$$

$$\tan^2 15^\circ + 2\sqrt{3}\tan 15^\circ - 1 = 0 \quad (5)$$

$$\tan 15^\circ = \frac{-2\sqrt{3} \pm \sqrt{12-4(1)(-1)}}{2(1)} \quad (5)$$

$$= -\sqrt{3} \pm 2$$

$$\tan 15^\circ > 0 \Rightarrow \tan 15^\circ = 2-\sqrt{3} \quad (5)$$

$\boxed{20}$

$$\tan 15^\circ = \frac{2\tan 7\frac{1}{2}^\circ}{1-\tan^2 7\frac{1}{2}^\circ} \quad (5)$$

$$2-\sqrt{3} = \frac{2t}{1-t^2}$$

$$(2-\sqrt{3})t^2 + 2t - (2-\sqrt{3}) = 0 \quad (5)$$

$$t = \frac{-2 \pm \sqrt{4+4(2-\sqrt{3})^2}}{2(2-\sqrt{3})} \quad (5)$$

$$= \frac{-1 \pm \sqrt{8-4\sqrt{3}}}{2-\sqrt{3}}$$

$$\tan 7\tfrac{1}{2}^\circ > 0$$

$$\therefore \tan 7\tfrac{1}{2}^\circ = \frac{-1+\sqrt{8-4\sqrt{3}}}{2-\sqrt{3}} \quad (5)$$

$$= \frac{-1+\sqrt{6}-\sqrt{2}}{2-\sqrt{3}} \quad (5) \qquad [\because (\sqrt{6}-\sqrt{2})^2 = 8-4\sqrt{3}]$$

$$= \frac{(\sqrt{6}-\sqrt{2}-1)(2+\sqrt{3})}{4-3} \quad (5)$$

$$= 2\sqrt{6}+\sqrt{18}-2\sqrt{2}-\sqrt{6}-2-\sqrt{3}$$

$$= \sqrt{6}-\sqrt{3}+\sqrt{2}-2. \quad (5)$$

$\boxed{35}$

(b)
$$\frac{\operatorname{cosec}A}{\operatorname{cosec}A-1} + \frac{\operatorname{cosec}A}{\operatorname{cosec}A+1}$$

$$= \frac{\operatorname{cosec}A(\operatorname{cosec}A+1)+\operatorname{cosec}A(\operatorname{cosec}A-1)}{\operatorname{cosec}^2A-1} \quad (5)$$

$$= \frac{2\operatorname{cosec}^2A}{\cot^2A} \quad (5)$$

$$= \frac{2\frac{1}{\sin^2A}}{\frac{\cos^2A}{\sin^2A}} \quad (5)\ (5)$$

$$= 2\sec^2A \quad (5)$$

$\boxed{25}$

(ii)
$$\left(1+\cos\tfrac{\pi}{8}\right)\left(1+\cos\tfrac{3\pi}{8}\right)\left(1+\cos\tfrac{5\pi}{8}\right)\left(1+\cos\tfrac{7\pi}{8}\right)$$

$$= \left(1+\cos\tfrac{\pi}{8}\right)\left(1+\cos\tfrac{3\pi}{8}\right)\left(1+\cos(\pi-\tfrac{3\pi}{8})\right)\left(1+\cos(\pi-\tfrac{\pi}{8})\right) \quad (5)$$

$$= \left(1+\cos\tfrac{\pi}{8}\right)\left(1+\cos\tfrac{3\pi}{8}\right)\left(1-\cos\tfrac{3\pi}{8}\right)\left(1-\cos\tfrac{\pi}{8}\right) \quad (5)$$

$$= \left(1-\cos^2\tfrac{\pi}{8}\right)\left(1-\cos^2\tfrac{3\pi}{8}\right) \quad (5)$$

$$= \sin^2\tfrac{\pi}{8}\sin^2\tfrac{3\pi}{8} \quad (5)$$

$$= \tfrac{1}{2}\left(1-\cos\tfrac{\pi}{4}\right)\tfrac{1}{2}\left(1-\cos\tfrac{3\pi}{4}\right) \quad (5)$$

$$= \tfrac{1}{4}\left(1-\tfrac{1}{\sqrt{2}}\right)\left(1+\tfrac{1}{\sqrt{2}}\right) \quad (5)$$

$$= \tfrac{1}{4}\left(1-\tfrac{1}{2}\right) = \tfrac{1}{8}.$$

$\boxed{30}$

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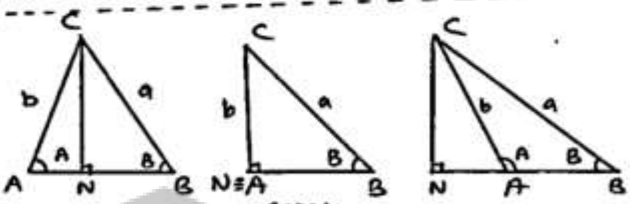
14. (a) In the usual notation for any triangle ABC, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. (5) $\boxed{5}$



Case (i)
$CN = b\sin A$
$CN = a\sin B$ (5)

Case (ii)
$CN = b = b\sin A$
$CN = a\sin B$ (5)

Case (iii)
$CN = b\sin(\pi-A) = b\sin A$
$CN = a\sin B$ (5)

$$CN = b\sin A = a\sin B$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Similarly, $\frac{\sin A}{a} = \frac{\sin C}{c}$ (5)

$$\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$\boxed{20}$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k \text{ (say)},$$

$$\frac{2b}{c+a-b}\cot\frac{B}{2}$$

$$= \frac{2k\sin B\cot B/2}{k\sin C + k\sin A - k\sin B} \quad (5)$$

$$= \frac{4\cancel{\sin\frac{B}{2}}\cos\frac{B}{2}\cos B/2}{\left[2\sin\left(\frac{C+A}{2}\right)\cos\left(\frac{C-A}{2}\right) - 2\sin\frac{B}{2}\cos\frac{B}{2}\right]\cancel{\sin B/2}} \quad (5)\ (5)\ (5)$$

$$= \frac{2\sin\left(\frac{C}{2}+\frac{A}{2}\right)}{\cos\left(\frac{C-A}{2}\right) - \cos\left(\frac{C+A}{2}\right)} \quad (5)\ (5)$$

$$= \frac{2\left(\sin\frac{C}{2}\cos\frac{A}{2} + \cos\frac{C}{2}\sin\frac{A}{2}\right)}{2\sin\frac{C}{2}\sin\frac{A}{2}} \quad (5)\ (5)$$

$$= \frac{\sin\frac{C}{2}\cos\frac{A}{2}}{\sin\frac{C}{2}\sin\frac{A}{2}} + \frac{\cos\frac{C}{2}\sin\frac{A}{2}}{\sin\frac{C}{2}\sin\frac{A}{2}}$$

$$= \cot\frac{A}{2} + \cot\frac{C}{2} \quad (5)$$

$$\Rightarrow (c+a-b)\left(\cot\frac{C}{2}+\cot\frac{A}{2}\right) = 2b\cot\frac{B}{2} \quad \boxed{45}$$

If $a+c=2b$,

$$(2b-b)\left(\cot\frac{C}{2}+\cot\frac{A}{2}\right) = 2b\cot\frac{B}{2} \quad (5)$$

$$\cot\frac{C}{2}+\cot\frac{A}{2} = 2\cot\frac{B}{2} \quad \boxed{5}$$

(b) (i)

$$\cos(\pi\cos x) = \cos(\pi\sin x)$$

$$\pi\cos x = 2k\pi \pm \pi\sin x, \ k\in\mathbb{Z} \quad (5)$$

$$\cos x \mp \sin x = 2k \quad (5)$$

$$\frac{1}{\sqrt{2}}\cos x \mp \frac{1}{\sqrt{2}}\sin x = \sqrt{2}k \quad (5)$$

$$\cos\frac{\pi}{4}\cos x \mp \sin\frac{\pi}{4}\sin x = \sqrt{2}k \quad (5)$$

$$\cos\left(x \pm \frac{\pi}{4}\right) = \sqrt{2}k \quad (5)$$

$$-1 \le \sqrt{2}k \le 1 \text{ and } k\in\mathbb{Z}$$

$$-\frac{1}{\sqrt{2}} \le k \le \frac{1}{\sqrt{2}} \text{ and } k\in\mathbb{Z}$$

$$\therefore k=0 \quad (5)$$

$$\cos\left(x+\frac{\pi}{4}\right) = \cos\frac{\pi}{2} \text{ or } \cos\left(x-\frac{\pi}{4}\right) = \cos\frac{\pi}{2} \quad (10)$$

$$x+\frac{\pi}{4} = 2n\pi \pm \frac{\pi}{2}, n\in\mathbb{Z} \text{ or } x-\frac{\pi}{4} = 2n\pi \pm \frac{\pi}{2}, n\in\mathbb{Z}$$

$$x = 2n\pi \pm \frac{\pi}{4}, n\in\mathbb{Z} \quad (5)$$

$$\text{or } x = 2n\pi \pm \frac{3\pi}{4}, n\in\mathbb{Z} \quad \boxed{45}$$

(ii) $\tan^{-1}\left(\frac{1}{3x-1}\right) - \tan^{-1}\left(\frac{1}{3x+1}\right) + \tan^{-1}2 = \frac{\pi}{2}$

Let $\alpha = \tan^{-1}\frac{1}{3x-1}$, $\beta = \tan^{-1}\frac{1}{3x+1}$, $\gamma = \tan^{-1}2$

$$\alpha - \beta = \frac{\pi}{2} - \gamma \quad (5)$$

$$\tan(\alpha-\beta) = \cot\gamma \quad (5)$$

$$\frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \frac{1}{2} \quad (5)$$

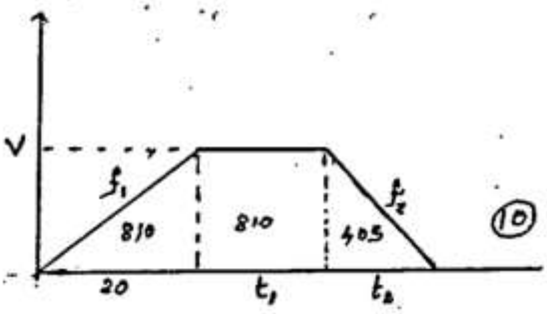
$$\frac{\frac{1}{3x-1} - \frac{1}{3x+1}}{1+\frac{1}{3x-1}\cdot\frac{1}{3x+1}} = \frac{1}{2} \quad (5)$$

$$\frac{2}{9x^2} = \frac{1}{2} \quad (5)$$

$$x^2 = \frac{4}{9}$$

$$x = \pm\frac{2}{3} \quad (5) \quad \boxed{30}$$

15 a)



(10)

i $\frac{1}{2} V \cdot 20 = 810$ (10)

$V = 81$ (5)

ii $f_1 = \frac{V}{20}$ (5)

$= \frac{81}{20}$

$= 4.05$ (5)

iii $405 = \frac{1}{2} \cdot V \cdot t_2$ (10)

$= \frac{1}{2} \cdot 81 \cdot t_2$

$t_2 = 10$ (5)

$f_2 = \frac{V}{t_2}$ (5)

$= \frac{81}{10}$

$= 8.1$ (5)

iv $810 = V \cdot t_1$ (10)

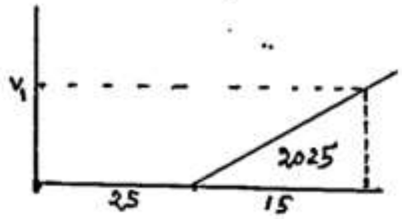
$= 81 \cdot t_1$

$t_1 = 10$ (5)

Total time $= 20 + 10 + 10$

$= 40$ (5)

v



$\frac{1}{2} V_1 \cdot 15 = 2025$ (10)

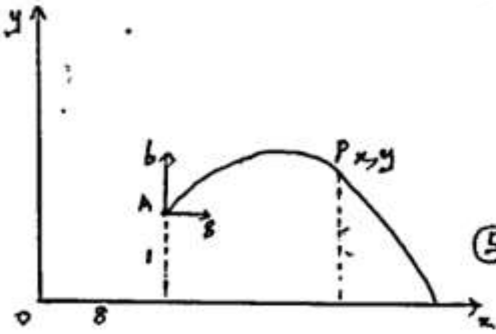
$V_1 = 270$ (5)

$f = \frac{V_1}{15}$

$= 18$ (5)

$\boxed{100}$

b)



(5)

$\rightarrow A \rightarrow P$ $\quad x - 8 = 8t$ (10)

$\uparrow$ $\quad y - 1 = bt - \frac{1}{2} g t^2$ (10)

$= b\left(\frac{x-8}{8}\right) - \frac{1}{2} g \left(\frac{x-8}{8}\right)^2$

$y = \frac{b}{8}\left(\frac{x-8}{1}\right) - \frac{g}{128}\left(x^2 - 16x + 64\right) + 1$ (10)

$= -\frac{g}{128} x^2 + \left(\frac{16b + 16g}{128}\right) x + \frac{128 - 64g}{128}$

$= \frac{1}{128}\left[-gx^2 + (16b + 16g)x + 128 - 64g\right]$

$= \frac{1}{128}\left[ax^2 + bx + c\right]$

$a = -g$, $b = 16b + 16g$, $c = 128 - 64g$

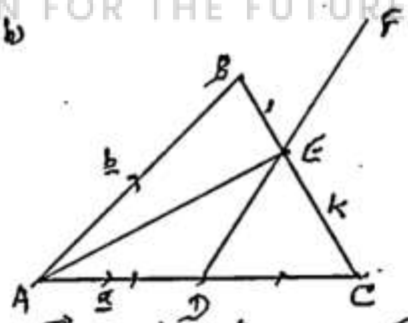
(5) (5) (5)

$\boxed{50}$

16 a) Theory (10)

$\boxed{10}$

b)



(10)

I $\vec{CB} = \vec{CA} + \vec{AB}$ (5)

$= -2a + b$ (5)

$\vec{CG} = \frac{k}{k+1} \vec{CB}$ (10)

$= \frac{k}{k+1}\left[b - 2a\right]$

$\vec{AE} = \vec{AC} + \vec{CE}$

$= 2a + \frac{k}{k+1}(b - 2a)$ —— (1) (10)

$= \frac{2}{k+1}a + \frac{k}{k+1}b$ (5)

II $\vec{DE} = \vec{DC} + \vec{CE}$

$= a + \frac{k}{k+1}(b - 2a)$ (10)

$= \frac{1-k}{k+1}a + \frac{k}{k+1}b$

$\vec{EF} = 7\vec{DE}$ (10)

$= 7\left[\frac{1-k}{k+1}a + \frac{k}{k+1}b\right]$

$\vec{AE} = \vec{AF} + \vec{FE}$

$= \vec{AF} - \frac{7}{k+1}[(1-k)a + kb]$ —— (2) (10)

(1) + (2) ⇒

$\frac{1}{k+1}[2a + kb] = \vec{AF} - \frac{7}{k+1}[(1-k)a + kb]$ (10)

$\vec{AF} = \frac{1}{k+1}[(9-7k)a + 8kb]$

III If A, B, F collinear

a) $\vec{AF} = \lambda\vec{AB}$ (10)

$\Rightarrow 9 - 7k = 0$

$k = \frac{9}{7}$ (5) $\boxed{100}$

b) $|a| = 3, |b| = 2, \cos B\hat{A}C = \frac{1}{3}$

I $a \cdot b = |a||b|\cos B\hat{A}C$ (5)

$= 3 \cdot 2 \cdot \frac{1}{3}$

$= 2$ (5)

II $\vec{BC} = 2a - b$

$\vec{AB} \cdot \vec{BC} = b \cdot (2a - b)$ (5)

$= 2a \cdot b - b^2$

$= 2 \cdot 2 - 2^2$ (5)

$= 0$

$AB \perp BC$

$A\hat{B}C = \frac{\pi}{2}$ (5)

$\vec{AD} \cdot \vec{DE} = a \cdot \left[a + \frac{k}{k+1}(b - 2a)\right]$ (5)

$= a \cdot [(1-k)a + kb]\frac{1}{k+1}$

$= \frac{1}{k+1}[(1-k)a^2 + k\, a \cdot b]$

$= \frac{1}{k+1}[(1-k)3^2 + k \cdot 2]$

$= \frac{9-7k}{k+1}$

$\neq 0$ (5) $\left(\because k \neq \frac{9}{7}\right)$

$AD \perp DE$

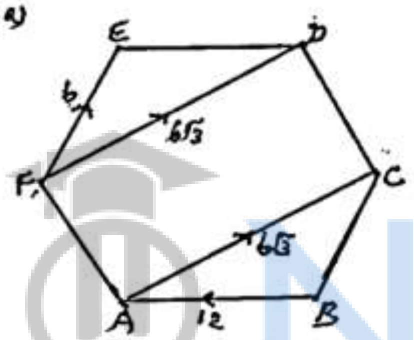
$C\hat{D}F = \frac{\pi}{2}$

$A\hat{B}C = C\hat{D}F$

B, D, C, F cyclic. (5)

$\boxed{140}$

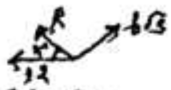
17 a)



I $R^2 = 12^2 + (6\sqrt3)^2 + 2 \cdot 12 \cdot 6\sqrt3 \cdot \cos 150°$ (10)

$= 36$

$R = 6$ (5)

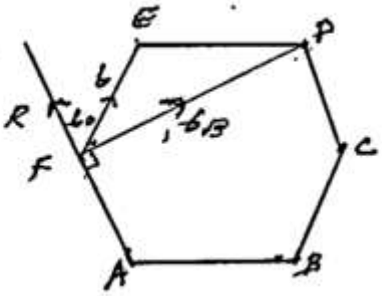


$\tan\alpha = \frac{6\sqrt3 \sin 30}{12 - 6\sqrt3 \cos 30}$ (5)

$= \sqrt3$

$\alpha = 60°$ (5)

R, acts along $\vec{AF}$



(10)

7

let R_1 be the resultant of R and $6\sqrt{3}$ along FB 10

$R_1^2 = R^2 + (6\sqrt{3})^2$ 5

$= 6^2 + (6\sqrt{3})^2$ 5

$R_1 = 12$

$\tan\beta = \frac{6\sqrt{3}}{6}$ 5

$= \sqrt{3}$ 5

$\beta = 60^\circ$

β is an angle between R_1 and R 5

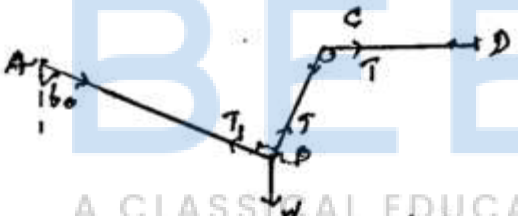
$\Rightarrow R_1$ acts along FE

Therefore Resultant of all forces is $R_1 + 6 = 12 + 6$

$= 18$ along $\vec{FE}$ 10

$\boxed{80}$

b)



10

I at the point B, using Lami's Theorem

$\frac{T_1}{\sin 150} = \frac{T}{\sin 120} = \frac{W}{\sin 90^\circ}$ 10

$\frac{T_1}{\frac{1}{2}} = \frac{T}{\frac{\sqrt{3}}{2}} = W$ 5

$T_1 = \frac{W}{2}$ 5

$T = \frac{W\sqrt{3}}{2}$ 5

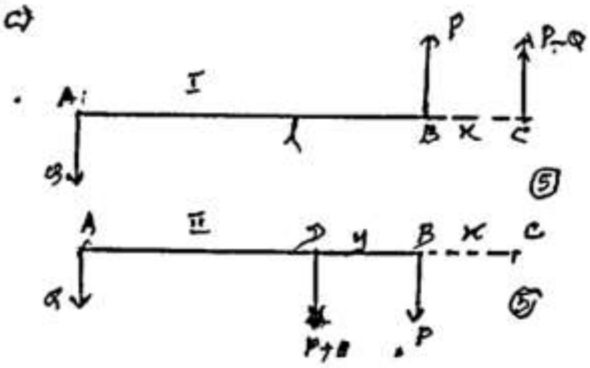
II Reaction at C

$R = 2T.\cos 60^\circ$ 5

$= W\frac{\sqrt{3}}{2}$ 5

$\boxed{35}$

c)



5

5

at I $\quad Px = Q(l+x)$ 10

$(P-Q)x = Ql$ —— (a)

at II $\quad Py = Q(L-y)$ 10

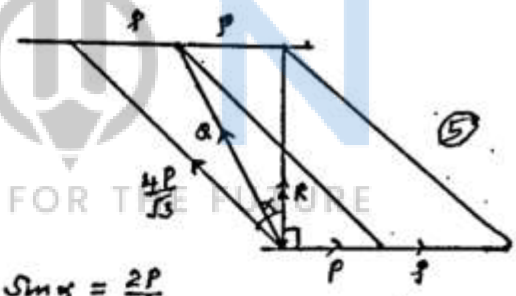
$(P+Q)y = Ql$ —— (b)

$x + y = \frac{Ql}{P-Q} + \frac{Ql}{P+Q}$ 5

$= \frac{Ql[P+Q+P-Q]}{P^2-Q^2}$

$= \frac{2PQl}{P^2-Q^2}$

$\boxed{35}$



5

$\sin\alpha = \frac{2P}{\frac{4P}{\sqrt{3}}}$

$= \frac{\sqrt{3}}{2}$ 5

$\alpha = 60^\circ$

Angle between forces 150° 5

$R = \frac{4P}{\sqrt{3}}\cos 60^\circ$ 5

$= \frac{2P}{\sqrt{3}}$

$Q^2 = P^2 + R^2$

$= P^2 + \left(\frac{2P}{\sqrt{3}}\right)^2$

$Q = \sqrt{\frac{7}{3}}\,P$ 5

$\boxed{25}$